Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 5, pp. 786–793, 2001

CALCULATION OF THE HYDRODYNAMIC INTERACTION OF BLADE CASCADES WITH CONSIDERATION FOR THE DIFFUSION OF UNSTEADY WAKES

V. A. Yudin

UDC 533.6.011

A semianalytical method was developed to calculate the hydrodynamic interaction of two blade cascades moving relative to one another. The potential perturbation of the flow by the cascades and the vortex perturbation due to blade edge wakes are taken into account. Along with the steady wakes caused by boundary layer separation from the blade cascades, allowance is made of the unsteady wakes separating from the blade trailing edges because of change in velocity circulation on them. The unsteady wakes are calculated with allowance for their diffusion in the presence of flow viscosity using approximate boundary-layer theory. The method is implemented as a program for calculating the unsteady hydrodynamic characteristics of blade cascades on a personal computer. Examples of calculation and a comparison with experiment are given.

Introduction. Calculation of the hydrodynamic interaction of cascades by numerical simulation of the complete Euler or Navier–Stokes equations is a difficult computational problem [1–4]. Available programs require continuous operation of the most powerful computers over tens or even hundreds of hours. Moreover, the calculation time increases considerably with decrease in blade-to-blade clearance and increase in the number of rotor and stator blades.

For the problem of potential cascade flow in a quasisteady formulation, Saren [5] proposed a semianalytical method of solution based on representation of the relative fluid velocity on blades in the form of a series in powers of a small parameter determined by the blade-to-blade clearance. Using this method in a linear approximation, Yudin [6] took into account the steady wakes caused by boundary layer separation from blade cascades and developed a program for routine calculations of unsteady hydrodynamic characteristics on personal computers [7]. The problem was also solved in a quasisteady formulation, i.e., ignoring the unsteady vortex wakes separating from blade cascades because of change in fluid velocity circulation. However, new experimental data [8] indicate that these wakes, diffusing into the flow, are a main cause of losses of total pressure and decrease in the efficiency of the cascade stages.

In the present paper, along with steady wakes, we take into account unsteady wakes (according to the Thomson theorem on the conservation of velocity circulation over fluid contours). Under the assumption of small intensity of these wakes, their diffusion is allowed for within the framework of boundary-layer theory. As in [5, 6], the problem reduces to a system of recurrent formulas for the coefficient of expansion of the relative fluid velocity over blade cascades. The program for calculating unsteady hydrodynamic characteristics does not require large expenditures of time and memory, and, hence, can be implemented on personal computers.

Basic Assumptions. In the plane of the complex variable $z = x + iy$, we consider a double-row blade cascade in an incompressible fluid flow having specified velocity $\boldsymbol{V}_{-\infty}$ at infinity ahead of the cascade. Let cascade No. 2, located downstream, move relative to cascade No. 1 with constant velocity $\boldsymbol{u}$ along the y axis (Fig. 1). We assume that the blade cascades are smooth and have sharp trailing edges.

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090.
Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 5, pp. 61–69, September–October, 2001. Original article submitted December 13, 2000; revision submitted February 23, 2001.

0021-8944/01/4205-0786 $25.00

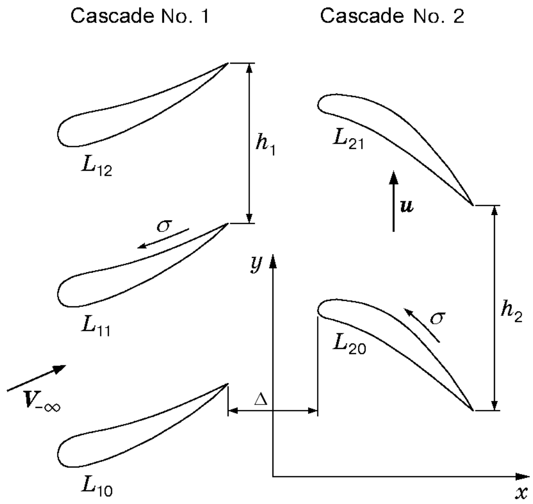

Fig. 1. Two-row blade cascade.

In the present formulation of the problem, the hydrodynamic interaction of the cascades is due to the following main factors: 1) potential perturbation, which propagates both downstream and upstream and is caused by deceleration and acceleration of the flow in the neighborhood of the blades of both cascades; 2) vortex perturbation due to boundary layer separation from the blades of cascade No. 1 (steady vortex wakes), which propagates only downstream and interacts with the blades of cascade 2; 3) perturbation due to the unsteady vortex wakes separating from the blade trailing edges of both cascades because of change in velocity circulation on them.

We consider small perturbations of the main flow by the blade cascades (the blades are rather thin, slightly curved, and are streamlined at small angles of attack). In this connection, we assume that the vortex wakes are located along the critical streamlines of the main flow through the cascades (the evolution of the wakes behind cascade No. 1 during passage through cascade No. 2 is neglected). In calculating the perturbation due to the unsteady vortex wakes behind cascade No. 2 and the perturbation upstream of the unsteady vortex wakes behind cascade No. 1, we simulate them by velocity discontinuity lines, as is done within the framework of the ideal fluid model. Calculations for the unsteady vortex wakes downstream of cascade No. 1 are performed with allowance for the diffusion of these wakes in the presence of flow viscosity [9]. The effect of the unsteady vortex wakes behind cascade No. 2 on the fluid flow in the region of cascade No. 1 is ignored.

Method of Solution. For each time t, the complex fluid velocity at the point z of the field flow is written as

$$V(z,t) = v(z,t) + J(z) + J_1(z,t) + J_2(z,t). \quad (1)$$

Here v is a function that is analytical in z everywhere outside the double-row cascade (v determines the potential perturbation of the flow by the cascades), J is a piecewise-continuous function that determines the complex fluid velocity behind cascade No. 1 in the absence of cascade No. 2 (J is generated by the steady vortex wakes behind cascade No. 1), and J_1 and J_2 correspond to the velocity fields produced by the unsteady wakes behind cascade Nos. 1 and 2. The function J_1 is analytical in z on the left half-plane $x < -\Delta/2$, and J_2 is analytical in z on the right half-plane $x > -\Delta/2$, except on the velocity discontinuity lines behind cascade No. 2 (Δ is the cascade spacing).

In representation (1), the function $v(z,t)$ should satisfy the following conditions:

1) The condition of nonpenetration of the fluid on the blade cascades

$$\mathrm{Im}_i\,\{v(z,t)\exp(i\alpha_\mu(\sigma))\} = \begin{cases} -\mathrm{Im}_i\,\{J_1(z)\exp(i\alpha_1(\sigma))\}, & z \in L_{1k}, \\ -u\cos\alpha_2(\sigma) - \mathrm{Im}_i\,\{J(z) + J_1(z,t) + J_2(z,t)\}, & z \in L_{2k}, \end{cases} \quad (2)$$

where μ is the cascade number, k is the blade cascade number, $L_{\mu k}$ is the contour of the kth blade of the μth cascade, $\alpha_\mu(\sigma)$ is the angle formed by the tangent to the blade $L_{\mu k}$ at the point z and in the positive direction of the x axis, and σ is the arc length of the blade contour reckoned from the trailing edge in the positive direction (counterclockwise);

2) The condition of constancy of the fluid velocity at infinity ahead of the cascades

$$\lim_{x\to-\infty} v(z,t) = V_{-\infty}; \tag{3}$$

3) The condition of periodicity of the flow in the y direction

$$v(z,t) = v(z+iL,t) = v(z,t+L/u), \tag{4}$$

where $L = N_1h_1 = N_2h_2$, h_1 and h_2 are the blade spacings for the cascades, and N_1 and N_2 are the numbers of blades in the periods of the double-row cascade;

4) Joukowski–Chaplygin condition of the finiteness of velocity at the sharp trailing edges of the blade cascades.

According to (3) and (4), the function $v(z,t)$ is defined by the Cauchy formula for periodic functions:

$$v(z,t) = \frac{1}{2Li}\int\limits_{L_1}\sum_{m=0}^{N_1-1} v_{1m}(\zeta_1,t)\Big[\coth\pi\Big(\frac{z-\zeta_1+\Delta/2}{L} - i\,\frac{m}{N_1}\Big)+1\Big]d\zeta_1$$

$$+\frac{1}{2Li}\int\limits_{L_2}\sum_{m=0}^{N_2-1} v_{2m}(\zeta_2,t)\Big[\coth\pi\Big(\frac{z-\zeta_2-iut-\Delta/2}{L} - i\,\frac{m}{N_2}\Big)+1\Big]d\zeta_2 + V_{-\infty}. \tag{5}$$

Here L_1 and L_2 are the initial blades of the first and second cascades displaced along the x axis by $\Delta/2$ and $-\Delta/2$, respectively, at the time $t=0$, $v_{\mu k}(\zeta_\mu,t) = v(z_k,t)$, $z_1 = \zeta_1 + ikh_1 - \Delta/2$ ($\zeta_1 \in L_1$), and $z_2 = \zeta_2 + ikh_2 + iut + \Delta/2$ ($\zeta_2 \in L_2$).

In the linear formulation of the problem considered, the influence functions of the unsteady vortex wakes $J_\mu(z_\mu,t)$ for the values $z_\mu = x_\mu + jy_\mu$ in the regions $x_1 < -\Delta/2$ and $x_2 > -\Delta/2$ are expressed by the integrals [10]

$$J_\mu(z_\mu,t) = \frac{1}{2Li}\int\limits_0^\infty\sum_{m=0}^{N_\mu-1}\gamma_{\mu m}(\zeta_\mu,t)\Big[\coth\pi\Big(\frac{z_\mu-\zeta_\mu(\tau)}{L} - i\,\frac{m}{N_\mu}\Big)+1\Big]d\tau, \tag{6}$$

where τ is the angular position on the velocity discontinuity line behind the blade L_μ reckoned from its trailing edge. The intensity $\gamma_{\mu m}(\zeta_\mu(\tau),t)$ of the unsteady vortex wake at the point $\zeta_\mu(\tau)$ at time t is defined by the formula

$$\gamma_{\mu m}(\zeta_\mu(\tau),t) = -\frac{1}{V_{0\mu}(\tau)}\,\frac{\partial\Gamma_{\mu m}}{\partial t}\Big|_{t=t_1} \qquad \Big(t = t_1 + T(\tau), \quad T(\tau) = \int\limits_0^\tau \frac{d\tau}{V_{0\mu}(\tau)}\Big). \tag{7}$$

Here $\Gamma_{\mu m}(t) = \int\limits_{L_\mu} V_{\mu m}(s)\,ds$ is the fluid velocity circulation around the mth blade of the μth cascade; $V_{0\mu}(\tau)$ is the relative velocity of the main steady flow (corresponding to $\Delta=\infty$) on the critical streamline behind the blade L_μ in a coordinate system attached to the μth cascade.

Following [5, 6], we write the relative fluid velocity $V_{\mu k}(s,t)$ as

$$V_{\mu k}(s,t) = \sum_{n=0}^\infty\sum_{r=0}^n [u_{\mu nr}(s)\cos r(\omega t + k\psi_\mu) + v_{\mu nr}(s)\sin(\omega t + k\psi_\mu)]\exp(-2\pi n\Delta/r), \tag{8}$$

where $\omega = 2\pi u/L$ and $\psi_\mu = (-1)^\mu 2\pi/N_\mu$.

In representation (5), let the point z tend from the flow field to the kth blade of the μth cascade. Using the limiting Plemelj–Sokhotskii formulas, expressions (6), and the nonpenetration condition (2) and expanding $\coth z$ in the power series

$$\coth z + 1 = \begin{cases} 2\displaystyle\sum_{n=0}^\infty \exp(-2nz), & \mathrm{Real}\, z > 0, \\ -2\displaystyle\sum_{n=1}^\infty \exp(2nz), & \mathrm{Real}\, z < 0, \end{cases}$$

we obtain the following system of recurrent formulas for the coefficients in the series expansion of the fluid velocity (8):

$$K_{1r}(U_{1nr}) = \Pi_{1nr}(U_{2pq},\ p \in \overline{1, n-1},\ q \in \overline{0, p}),$$
$$K_{2r}(U_{2nr}) = \Pi_{2nr}(U_{1pq},\ p \in \overline{1, n},\ q \in \overline{0, p}). \tag{9}$$

Here $U_{\mu nr} = u_{\mu nr} + jv_{\mu nr}$ ($\mu = 1,\, 2;\ i \neq j$).

In system (9), the integral operator $K_{\mu r}$ has the form

$$K_{\mu r}(U_{\mu nr}) = \frac{1}{2}\, U_{\mu nr}(\sigma) - \frac{\exp\,(i\alpha^{\mu}(\sigma))}{2Li} \int\limits_{L_\mu} U_{\mu nr}(s) \sum_{m=0}^{N_\mu - 1} \exp\,(-jrm\psi_\mu)$$
$$\times \Big[\coth \pi\Big(\frac{z_\mu - \zeta_\mu}{L} - i\,\frac{m}{N_\mu}\Big) + 1 - R_{\mu r}(\sigma)\Big]\, ds,$$

where

$$R_{\mu r}(\sigma) = \int\limits_0^\infty \frac{jr\omega t}{V_{0\mu}(\tau)} \exp\,(j\omega r T(\tau)) \Big[\coth \pi\Big(\frac{z_\mu - \zeta_\mu(\tau)}{L} - i\,\frac{m}{N_\mu}\Big) + 1\Big]\, d\tau.$$

The right sides of Eqs. (9) $\Pi_{\mu nr}$ are defined by the equalities

$$\Pi_{100}(\sigma) = V_{-\infty} \exp\,(i\alpha_1(\sigma)),$$

$$\Pi_{200}(\sigma) = \frac{\exp\,(i\alpha_1(\sigma))}{h_1 i} \int\limits_{L_1} u_{100}(s)\, ds + (V_{-\infty} + iu) \exp\,(i\alpha_2(\sigma)) + K_{20}[A_0(\sigma)],$$

$$\Pi_{\mu nr}(\sigma) = (-1)^{\mu} \delta_2 æ_{\nu r} \exp\,(i\alpha_\mu(\sigma)) \Bigg[(1 - ij) \sum_{k=æ_1}^{n-E(\nu/2)} P_{\nu k}^{n-k}(k - n + r)\varphi_\mu(\sigma, n-k)$$
$$+ (1 + ij) \sum_{k=æ_2}^{n-E(\nu/2)} P_{\nu k}^{n-k}(k - n - r)\overline{\varphi}_\mu(\sigma, n-k)\Bigg] + (2 - \nu) K_{2r}[A_r(\sigma)],$$

$$\varphi_\mu(\sigma, k) = \exp\,[(-1)^{\nu} 2\pi k\, (\xi_\mu(\sigma) + i\eta_\mu(\sigma))\,/L],$$

$$P_{\mu n}^{k}(r) = \begin{cases} \dfrac{1}{2h_\mu i} \displaystyle\int\limits_{L_\mu} [U_{\mu nr}(\sigma) - (2 - \nu) A_{nr}(\sigma)] \varphi_\mu(\sigma, k)\, d\sigma, & r > 0, \\ \dfrac{1}{h_\mu i} \displaystyle\int\limits_{L_\mu} [U_{\mu n0}(\sigma) - (2 - \nu) A_{n0}(\sigma)] \varphi_\mu(\sigma, k)\, d\sigma, & r = 0, \\ \dfrac{1}{2h_\mu i} \displaystyle\int\limits_{L_\mu} [\bar{U}_{\mu nl}(\sigma) - (2 - \nu) \bar{A}_{nl}(\sigma)] \varphi_\mu(\sigma, k)\, d\sigma, & l = -r > 0, \end{cases}$$

$$A_{nr}(\sigma) = \delta_{nr} f_r(\sigma) \exp\,(i\alpha_2(\sigma)) \exp\,(2\pi n\Delta/L) + J_{1nr}(\sigma) \exp\,(i\alpha_2(\sigma)),$$

$$æ_1 = E\Big(\frac{n - r + 1}{2}\Big), \qquad æ_2 = E\Big(\frac{n + r + 1}{2}\Big),$$

$$æ_{\mu r} = \frac{1}{N_\mu} \sum_{m=0}^{N_\mu} \exp\Big(\pm \frac{i2\pi rm}{N_\mu}\Big) = \begin{cases} 1, & r = r_1 N_\mu, \\ 0, & r \neq r_1 N_\mu, \end{cases} \quad r_1 = 0, 1, 2, \ldots,$$

$$\nu = \begin{cases} 2, & \mu = 1, \\ 1, & \mu = 2, \end{cases} \qquad \delta_k = \begin{cases} 0.5, & k = 0, \\ 1, & k \neq 0, \end{cases} \qquad \delta_{nr} = \begin{cases} 0, & n \neq r, \\ 1, & n = r. \end{cases}$$

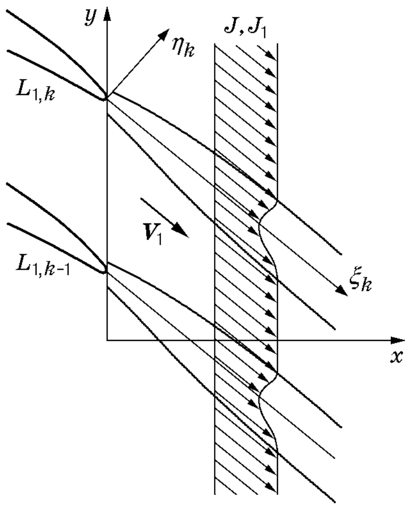

Fig. 2. Blade edge wakes.

Here $E(x)$ is the whole part of the number x; the bar denotes quantities that are complex conjugate in j.

The functions $f_r(\sigma)$ and $J_{1nr}(\sigma)$ in the expression for $A_{nr}(\sigma)$ are the coefficients of the series expansions of the functions $J_m(\sigma, t)$ and $J_{1m}(\sigma, t)$:

$$J_m(\sigma, t) = J\Big(z_2(s) + imh_2 + iut + \frac{\Delta}{2}\Big) = \sum_{r=0}^{\infty} f_r(\sigma) \exp\left(-jr(\omega t + m\psi_2)\right); \tag{10}$$

$$J_{1m}(\sigma, t) = J_1\Big(z_2(s) + imh_2 + iut + \frac{\Delta}{2}\Big) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} J_{1nr}(\sigma) \exp\left(-jr(\omega t + m\psi_2)\right) \exp\Big(-\frac{2\pi n\Delta}{L}\Big). \tag{11}$$

The difference between the expressions obtained and those given in [6] is due to the addition of the terms $R_{\mu r}(\sigma)$ to the kernels of the operators $K_{\mu r}$. These terms are due to the upstream effect of the unsteady vortex wakes behind the cascades and to the fact that the expressions for $\Pi_{\mu nr}(\sigma)$ contain additional terms defined by the function J_{1nr}, which arises when allowance is made for the diffusion of the unsteady wakes downstream of the first cascade.

Definition of the Functions $J(z)$ and $J_1(z, t)$. To define the function $J(z)$, we use the known dependence of the flow rate in the wake behind a single cascade (in the absence of cascade No. 2) on the coefficient of blade losses $\zeta_{\rm bl}$ (Fig. 2). Expressions for the coefficient f_r of expansion (10) are given in [6].

To define the function $J_1(z, t)$, we use the approximate solution of the problem of diffusion of a vortex sheet [9] (Fig. 2):

$$J_1(z, t) = \frac{V_1}{\sqrt{\pi}}\left[\Phi\left(\sqrt{\frac{\rm Re}{4h_1\xi_k}}\,\eta_k\right) - \frac{\sqrt{\pi}}{2}\,{\rm sign}\,\eta_k\right] \frac{\partial\Gamma_{1k}}{\partial t_1}\Big|_{t_1 = t - \xi_k/V_1}. \tag{12}$$

Here $\Phi(\theta) = \int\limits_0^\theta \exp(-\theta^2)\,d\theta$ is the probability integral, ${\rm Re} = V_1 h_1/\nu$ is the Reynolds number, ν is the kinematic viscosity, V_1 is the complex fluid velocity at infinity ahead of the cascade, and $\eta_k + i\xi_k = z\exp(i\alpha)$, where $\alpha = \arg V_1$ and $k = 0, \pm1, \pm2, \ldots$.

Expression (11) for the coefficients $J_{1nr}(\sigma)$ is obtained from (12) by a Fourier series expansion of J_1 in the variable y using (7) and (8) with conversion to the coordinate system of the second cascade. Finally, we have

$$J_{1nr}(\sigma) = \sum_{p=\beta_1}^{\beta_2} B_{p,\,n-|p|,\,r-p} \exp\left(-jp\omega_1 y_2(\sigma)\right) \exp\Big(\frac{2\pi|p|\Delta}{L}\Big), \tag{13}$$

where

$$\beta_1 = \Big[\frac{-n+r+1}{2}\Big], \qquad \beta_2 = \Big[\frac{n+r}{2}\Big], \qquad \omega_1 = \frac{2\pi}{L},$$

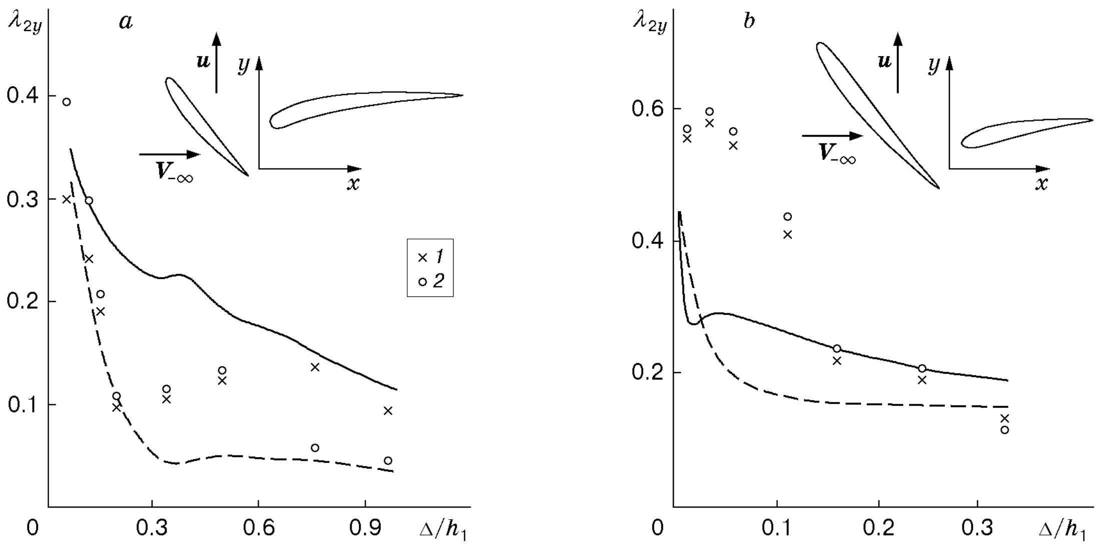

Fig. 3. Level of exciting forces versus axial clearance: solid curves refer to calculations by the proposed model, dashed curves refer to calculations by the model of a quasisteady flow [6], points are data of the experiment of [11] for $n_1 = 30.0$ (1) and 23.34 sec^{-1} (2): (a) $N_1 = 9$, $N_2 = 10$, $\tau_1 = 0.71$, $\tau_2 = 1.33$, and $\zeta_{\rm bl} = 0.021$; (b) $N_1 = 3$, $N_2 = 10$, $\tau_1 = 0.64$, $\tau_2 = 1.33$, and $\zeta_{\rm bl} = 0.04$.

$$B_{p,k,l} = -\frac{\delta_2 æ_{1r}}{h_1} jl\omega\Gamma_{1,k,|l|} \int A(x_2(\sigma)+\Delta, y) \exp\Big(\frac{jl\omega}{V_1}[(x_2(\sigma)+\Delta)\cos\alpha - y\sin\alpha]\Big) \exp(jp\omega_1 y)\, dy,$$

$$A(x,y) = \frac{1}{\sqrt{\pi}}\left[\Phi\left(\sqrt{\frac{\rm Re}{4h_1\xi_0}}\,\eta_0\right) - \frac{\sqrt{\pi}}{2}\,{\rm sign}\,\eta_0\right];$$

Γ_{1nr} are the coefficients of the expansion of the circulation $\Gamma_{1m}(s)$ in series (8):

$$\Gamma_{1m}(s) = \sum_{n=0}^{\infty}\sum_{r=0}^{n} \Gamma_{1nr} \exp(-jr(\omega t + m\psi_1)) \exp\Big(\frac{-2\pi n\Delta}{L}\Big).$$

We note that, by virtue of the properties of the function Φ, the quantity A decreases rapidly with distance from the wake axis $[(y_0, x_2(\sigma)+\Delta)$, where $y_0 = -(x_2(\sigma)+\Delta)/{\rm tg}\,\alpha]$, and to calculate the coefficients B_{pkl} approximately, it suffices to perform integration over y in a narrow interval $(y_0 - \varepsilon, y_0 + \varepsilon)$, where ε determines the width of the unsteady wake in the y direction.

Thus, from formulas (9)–(13), we can sequentially determine the coefficients $U_{\mu nr}$ of the expansion of the relative velocity $V_{\mu m}$ on the blades in series (8). The blade pressure, total forces, and the moment are further obtained using the Cauchy–Lagrange integral.

Program and Calculation Results. The above method is implemented in the form of a program for calculating the unsteady aerodynamic characteristics of blade cascades. The program is written in FORTRAN, and the time of calculation of one version on a Pentium-II computer is 1–5 min, depending on the computation version and the number of points of division of the initial blades. We note that the calculation time depends weakly on the number of blades in the general period of cascades (in contrast to methods of direct calculation of the Euler or Navier–Stokes equations).

Figure 3 shows results of our calculation and data of the experiment of [11]. The level of exciting forces on the blades of the second cascade are plotted on the ordinate:

$$\lambda_{2y} = (\max Y_2(t) - \min Y_2(t))/Y_2^0, \qquad t \in [0, T_2].$$

Here $T_2 = 2\pi h_1/u$ and $Y_2(t)$ is the circumferential component of the force on the blades and Y_2^0 is its mean value.

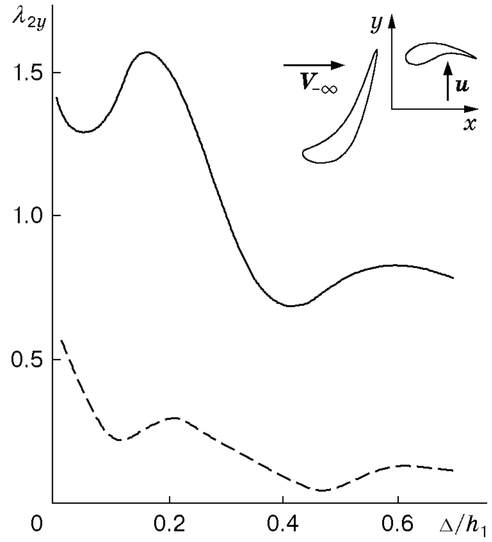

Fig. 4. Level of exciting forces versus axial clearance for large deflection angle of the first cascade ($N_1 = N_2 = 1$, $\tau_1 = \tau_2 = 1$, $\zeta_{\rm bl} = 0.03$, and $q = u/V_{-\infty} = 3.595$): the solid curve refers to calculation for the proposed model and the dashed curve refers to calculation for the quasisteady flow model [6].

An analysis of the curves in Fig. 3 shows that the unsteady vortex wakes make a considerable contribution to the level of unsteady exciting forces on the blades. Comparison of the calculation results and the experimental data of [11] shows that in Fig. 3a, the experimental data for various speeds of rotation of the rotor n_1 differ greatly and calculations using the proposed model are in good agreement with experiment only for $n_1 = 23.34$ sec^{-1}. In Fig. 3b, the observed data for various values of n_1 practically coincide. Calculations for the proposed model are in better quantitative agreement with the experiment than calculations using the model of a quasisteady flow [6]. For moderate axial clearances ($\Delta/h_1 \approx 0.1$–0.3) and for small clearances ($\Delta/h_1 < 0.1$), the calculations "trace" the qualitative behavior of the unsteady force, though at a different quantitative level. We note that in the literature, except in [11], there are no experimental data on the unsteady forces on row blades moving relative to one another.

An important feature of behavior of unsteady forces during interaction of cascades is the experimental nonmonotonic dependence of the values of λ_{2y} on the axial clearance for large deflection angles of the first cascade. In [12], this phenomenon is explained by superimposition of the potential perturbation and vortex perturbation (due to steady vortex wakes J) of the flow using a model of quasisteady flow around cascades. Since in the unsteady flow model considered, the zones of perturbed velocity due to steady J and unsteady J_1 vortex wakes practically coincide, the dependence λ_{2y} on the axial clearance should be nonmonotonic in the same cases as in the model of quasisteady flow around cascades. A comparison of the curves in Fig. 4 shows that the range of values of the axial clearance Δ in which the level of exciting forces λ_{2y} is nonmonotonic is almost the same in calculations using the proposed model and the quasisteady flow model. The quantitative difference is explained by the fact that pressure calculations for the quasisteady model ignore the term $\partial\varphi/\partial t$, which in the present calculation version makes a considerable contribution because of the large deflection angle of cascade No. 1 ($q = u/V_{-\infty} = 3.595$ and the Strouchal number $\mathrm{Sh} = \omega L/V_{-\infty} = 2\pi q$, which characterizes the unsteadiness level, is large).

The main result of the present work is the development of a program that takes into account the diffusion of unsteady vortex wakes behind blade cascades and can be useful in routine calculations of aerodynamic characteristics of blades on personal computers. In further work, we are planning to use the program to calculate a system of three cascades stator–rotor–stator and to compare the flow pressure and velocity fields with experiment [8].

The author is grateful to V. É. Saren for the proposed idea of solving the problem in the present formulation and for continuous attention to the work.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 00-02-17226) within the framework of the Integration Project No. 1 of the Siberian Division of the Russian Academy of Sciences.

REFERENCES

1. P. Cizmas and R. Subramanya, “Parallel computation of rotor–stator interaction,” in: *Unsteady Aerodynamics and Aeroelastisity of Turbomachines*, Proc. of the 8th Int. Symp. (Stockholm, Sweden, Sept. 14–18, 1997), Kluwer Acad. Publ., Dordrecht, etc. (1998), pp. 633–645.
2. V. Gnesin and R. Rzadkowski, “The 3D unsteady aerodynamic forces: The forced vibration of blades discs,” *ibid.*, pp. 613–632.
3. M. M. Rai, “Navier–Stokes simulation of rotor–stator interaction using patched and overlaid grids,” AIAA Paper No. 85-1519, Cincinatti, Ohi (1995).
4. V. É. Saren, N. M. Savin, D. J. Dorney, and R. M. Zacharias, “Experimental and numerical investigation of unsteady rotor–stator interaction influence on axial compressor stage (with IGV) performance,” in: *Unsteady Aerodynamics and Aeroelasticity of Turbomachines*, Proc. of the 8th Int. Symp. (Stockholm, Sweden, Sept. 14–18, 1997), Kluwer Acad. Publ., Dordrecht, etc. (1998), pp. 407–426.
5. V. É. Saren, “On the hydrodynamic interaction of blade cascades in a potential flow,” *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 4, 75–84 (1971).
6. V. A. Yudin, “Calculation of hydrodynamic interaction of blade cascades taking into account edge wakes,” *Tr. TsIAM*, No. 953, 52–67 (1981).
7. V. A. Yudin, “Calculation of interaction of blade cascades taking into account edge wakes in an ideal incompressible fluid,” *Tr. TsIAM*, No. 1127, 245–248 (1985).
8. N. M. Savin and V. É. Saren, “Hydrodynamic interaction of rows the stator-rotor–stator system of an axial turbomachine,” *Izv. Ross. Akad. Nauk, Mekh Zhidk. Gaza*, No. 3, 145–158 (2000).
9. H. Lamb, *Hydrodynamics*, Cambridge Univ. Press (1932).
10. D. N. Gorelov, V. B. Kurzin, and V. É. Saren, *Aerodynamics of Cascades in Unsteady Flow* [in Russian], Nauka, Novosibirsk (1974).
11. T. Adachi, K. Fukusado, N. Takahasi, and Y. Nakamoto, “Study of the interference between moving and stationary blade rows in axial flow blower,” *Bull. JSME*, **17**, No. 109, 904–911 (1974).
12. V. É. Saren and V. A. Yudin, “Effect of axial clearance on exciting forces in blade cascades,”in: *Aeroelasticity of Turbomachines* (collected scientific papers) [in Russian], Inst. of Hydrodynamics, Sib. Div., USSR Acad. of Sci., Novosibirsk (1984), pp. 33–42.